$1-p=1-2^4/6^9=\frac{10077680}{10077696}=P$, chance that D will throw less than 52.

P^2=chance that D and E will both throw less.

$\therefore$ aP^2=C's expectation on the supposition that C wins and no ties.

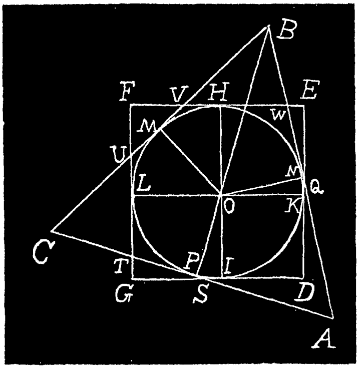
49. Proposed by B. F. FINKEL, A. M., M. Sc., Professor of Mathematics and Physics, Drury College, Springfield, Missouri.

A square whose side is $2a$ and an equilateral triangle whose altitude is $3a$ are fastened together at their centers, but otherwise free to move. If they are thrown on a floor at random, what is the average area common to both ?

Solution by HENRY HEATON, M. Sc., Atlantic, Iowa, and the PROPOSER.

In the figure let O be the common center of the square and the triangle.

Then $OK=ON=OH=OM=OL=OP=OI=a$.



Let the triangle $KON=2\theta$.

Then $\angle NOH=\frac{1}{2}\pi-2\theta$, $\angle HOM=\frac{2}{3}\pi-(\frac{1}{2}\pi-2\theta)=\frac{1}{6}\pi+2\theta$, $\angle MOL=\frac{1}{3}\pi-(\frac{1}{6}\pi+2\theta)=\frac{1}{6}\pi-2\theta$, $\angle LOP=\frac{2}{3}\pi-(\frac{1}{6}\pi-2\theta)=\frac{1}{2}\pi+2\theta$, and $\angle POI=\frac{1}{2}\pi-(\frac{1}{3}\pi+2\theta)=\frac{1}{6}\pi-2\theta$.

Area of surface, $KONQ$, $=a^2\tan\theta$;

area of surface, $NOHW$, $=a^2\tan(\frac{1}{4}\pi-\theta)$;

area of surface, $HOMV$, $=a^2\tan(\frac{1}{12}\pi+\theta)$;

area of surface, $MOLU$, $=a^2\tan(\frac{1}{6}\pi-\theta)$;

area of surface, $KOPT$, $=a^2\tan(\frac{1}{6}\pi+\theta)$;

area of surface, $POIS$, $=a^2\tan(\frac{1}{12}\pi-\theta)$;

area of square, $OKDI$, $=a^2$.

Hence the area common to the square and triangle is

$$S=a^2[1+\tan\theta+\tan(\tfrac{1}{4}\pi-\theta)+\tan(\tfrac{1}{12}\pi+\theta)+\tan(\tfrac{1}{6}\pi-\theta)+\tan(\tfrac{1}{6}\pi+\theta)+\tan(\tfrac{1}{12}\pi-\theta)].$$

The positions for $\theta>\frac{1}{12}\pi$ are exact repetitions of those for $\theta<\frac{1}{12}\pi$.

Hence the required average area is

$$\Delta=\int_0^{\frac{1}{12}\pi}Sd\theta\div\int_0^{\frac{1}{12}\pi}d\theta=\frac{12a^2}{\pi}\int_0^{\frac{1}{12}\pi}\Big[1+\tan\theta+\tan(\tfrac{1}{4}\pi-\theta)+\tan(\tfrac{1}{12}\pi+\theta)+$$

$$\tan(\tfrac{1}{6}\pi-\theta)+\tan(\tfrac{1}{6}\pi+\theta)+\tan(\tfrac{1}{12}\pi-\theta)\Big]d\theta=a^2\Big[1+\frac{12}{\pi}\log_e 2\Big].$$

This problem was also solved in a very excellent manner by G. B. M. Zerr.

50. Proposed by G. B. M. ZERR, A. M., Ph. D., Texarkana, Arkansas.

Find (1), the average length of all straight lines having a given direction, between 0 and a; (2), the average length of chords drawn from one extremity of the diameter a of a semi-circle to all points in the semi-circumference; and (3), find the average area of all triangles formed by a straight line of constant length a sliding between two straight lines at right angles.